LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SIXTH SEMESTER – **APRIL 2015**

MT 6606 - COMPLEX ANALYSIS

Date : 15/04/2015 Dept. No. Max. : 100 Marks
Time : 09:00-12:00

PART - A

Answer ALL questions. Each question carries 2 marks. (10 x 2 = 20 marks)

1. Show that the function $f(z) = \operatorname{Re} z$ is nowhere differentiable.
2. When do we say that a function u(x,y) is harmonic.
3. Find the points where the mapping $w = z + \frac{1}{z}$ is conformal. Also find the critical points.
4. Define a bilinear transformation.
5. Evaluate $\int_c \frac{dz}{z}$ where c is the circle $|z| = r$ described in the positive sense.
6. State Cauchy's inequality.
7. Expand cos z by Taylor's series about z=0.
8. Define essential singularity with an example.
9. Write down the formula for evaluating the residue at a pole of order m.
10. Calculate the residue of $\frac{z+1}{z(z-2)}$ at its poles.

PART - B

Answer any FIVE questions. Each question carries 8 marks. (5 x 8 = 40 marks)

11. Prove that the function $f(z) = \begin{cases} \frac{x^3(1+i) - y^3(1-i)}{x^2+y^2}; z \neq 0 \\ 0; z = 0 \end{cases}$ satisfies C-R equations at the origin but $f'(0)$ does not exist.
12. Show that $u = 2x - x^3 + 3xy^2$ is harmonic and find its harmonic conjugate.
13. Prove that any bilinear transformation can be expressed as a product of translation, rotation, magnification or contraction and inversion.
14. Evaluate $\int_c |z|\bar{z}dz$ where c is the closed curve consisting of the upper semicircle $|z| = 1$ and the line segment $-1 \leq x \leq 1$.
15. State and prove the Maximum Modulus theorem.
16. Expand $f(z) = \frac{z}{(z-1)(2-z)}$ in a Laurent's series valid for (i) $1 < |z| < 2$, (ii) $|z| > 2$.
17. State and prove the Fundamental theorem of algebra.
18. Using contour integration along the unit circle ,show that
$$\int_0^{2\pi} \frac{d\theta}{5 + 4\sin\theta} = \frac{2\pi}{3}.$$

PART - C

Answer any TWO questions. Each question carries 20 marks. (2 x 20 = 40 marks)

19. a) State and prove the sufficient conditions for f(z) to be differentiable at a point.

b) Find the analytic function f(z) = u+iv if $u+v=\dfrac{\sin 2x}{\cosh 2y-\cos 2x}$. (10+10)

20. a) State and prove Cauchy's integral formula.

b) Evaluate $\int_c \dfrac{\sin \pi z^2+\cos \pi z^2}{(z-1)(z-2)}dz$ where c is the circle $|z|=3$. (12+8)

21. a) State and prove Laurent's series.

b) State and Prove Rouche's Theorem. (12+8)

22. a) State and prove Cauchy Residue theorem.

b) By contour integration , show that $\int_0^\infty \dfrac{dx}{1+x^4}=\dfrac{\pi}{2\sqrt{2}}$. (8+12)

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